Exam
Physics of Lasers 2014
Apr 1, 2014 14:00-17:00
Room 5118.-156

- *Write only your student number on each exam sheet*
- *Write the answers in the space provided on these sheets*
- *Read the questions carefully, and give complete answers*
- *The exam consists of 6 questions; maximum score for each question is 10 point*

$$u(x,y,z) = \frac{-Ai}{\frac{ikw_0^2}{2} + z} \cdot \exp\left(-\frac{r^2}{w^2(z)}\right) \cdot \exp\left[-ikz\left(1 + \frac{2r^2}{k^2 w_0^4 + 4z^2}\right)\right]$$

$$w^2(z) = w_0^2 + \frac{4z^2}{k^2 w_0^2}$$

$$R(z) = z\left(1 + \frac{k^2 w_0^4}{4z^2}\right)$$

1a. The radius of curvature of a Gaussian laser beam can be determined by considering the solution to the parabolic equation as we did in class. Write down the term in the solution to the propagation equation that provides the radius of curvature as a function of distance (1pt). Draw a graph of the radius of curvature as a function of propagation distance (2pts). (Bare in mind that in our definition from class we defined w = w_0 when z = 0.)

1b. By taking a derivative of the radius of curvature with respect to distance, find the distance along the propagation direction where the radius or curvature is minimum (3pts).

1c. The answer with which you arrived above is a very important length scale in Gaussian beam optics and is referred to as the confocal parameter. What is the beam radius or diameter of the electric field at a distance equal to the confocal parameter (4pts) (note: I am not asking for the *radius of curvature*, but rather the physical beam radius)?

2a. Consider a resonator constructed using a two concave (R>0) mirrors of equal radius of curvature separated by a distance L. Write the two stability parameters for this two-mirror arrangement (1pt). What are the limits on L such that we have a stable resonator (4pts)?

2b. We similarly saw in class that we could use ABCD matrices to arrive at the same stability criteria. Draw a simple two mirror cavity and write down, in appropriate order, the ABCD matrices that will lead us to the famous (A+D+2)/4 stability result (1pt). (Note: you are not being asked to multiply the matrices.) The following two matrices may be of assistance:

$$\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix}$$

2c. The waste size in a resonator is given by the following expression:

$$w_0^2 = \frac{\lambda L}{\pi} \frac{\sqrt{(1 - g_1 g_2) g_1 g_2}}{g_1 + g_2 - 2 g_1 g_2}$$

Find an approximate expression for the beam waste of a confocal resonator (g_i=0) (2pts).

2d. Using the expression for the beam waste given in question 1, find an expression for the waste size at the mirror position (3pts) (The definitions of problem 1 may be of assistance.)

3a. As we learned in class, there exists a Fourier Transform relation between the temporal extent of a waveform in time and its frequency components:

$$f(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E(t)e^{-i\omega t}\,dt$$

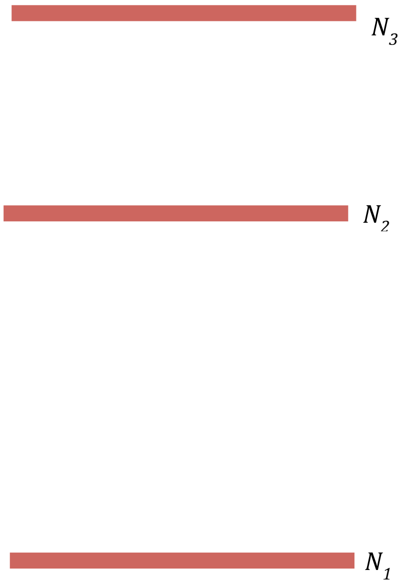
Calculate the Power Spectrum $|f(\omega)|^2$ of a wave train given by (5pt):

$$E(t) = \exp(i\omega_0 t) \quad for \quad -\frac{\tau_0}{2} < t < \frac{\tau_0}{2}$$

3b. Sketch the function you arrived at above (1pts).

3c. We can recognize the width of the power spectrum above to be inversely proportional to the 'time' over which the ω_0 oscillates. We could also frame this in the language of temporal coherence, namely, the spectral width and the coherence time are inversely related. Beginning with the relation $f\lambda = c$, calculate the coherence time of a light source centered at 500nm with a 10nm bandwidth (4pts).

4a



Consider this three level lasing system. Write down the rate equations for the three levels, without considering the lasing transition and energy density in the cavity. In doing so, consider all the mechanisms that result in transfer of populations from one level to another. Label them in the figure (3pts).

4b. The population inversion in such a three level laser system can be calculated based on the above equations to be:

$$\Delta N = N\frac{\left(1-\frac{\gamma_{32}}{\gamma_{21}}\frac{\Gamma/\gamma_{32}}{1+\Gamma/\gamma_{32}}\right)}{1+\frac{\Gamma/\gamma_{32}}{1+\Gamma/\gamma_{32}}\left(\frac{\gamma_{32}}{\gamma_{21}}+1\right)}$$

The threshold population inversion is then a complicated function of both the various relaxation rates and the pumping rate Γ. Find an expression for the pumping rate that leads to a population inversion (3pts). (note the population inversion in our notations is when $\Delta N<0$)

4c. Sketch a couple of curves for the population ΔN in this three level system. Consider a few cases of

$$\frac{\gamma_{32}}{\gamma_{21}}$$

and sketch them as a function of Γ/γ_{32} (2pts).

4d. Describe how we obtain a population inversion in a three level lasing system. Your description should include a discussion about the relative transition rates and how one achieves an inverted population between the lasing levels (2pts).

5a. One of the criteria for making a laser resonator is that an integer number of wavelengths must fit inside it (nodes at the mirrors). Write down the possible frequencies of oscillation of a resonator whose length is L (2pts).

5b. What is the "mode separation", the distance in frequency between modes (2pts)?

5c. The width of the resonance is determined by the mirror reflectivity to be

$$\delta\omega = \frac{c}{L}\frac{1-R}{\sqrt{R}}$$

sketch this function for a single cavity resonance as a function of mirror reflectivity R (2pt). What happens as R approaches 100% (2pt)?

5d. Within 5%, what is the reflectivity of a silver mirror at normal incidence for visible radiation (1pt)? Describe one method for obtaining a reflectivity approaching 100%? (1pt) (in case you need them, for silver n=0.07 and k=4.2)

6a. The practicum had us construct a laser from scratch. What type of laser was it (1pt)?

6b. what was lasing wavelength (1pt)? What was the pumping wavelength (1pt)? (a range of wavelengths will suffice)

6c. Our lasing material was a four level system, which in principle shouldn't have a threshold for population inversion. Yet in the experiment we saw a threshold in output power. What is the difference between these two concepts (2pts)? Draw a curve for steady state output power as a function of pumping rate (1pt).

6d.

$$\boxed{\omega = \omega_c \frac{\omega_0 + Q_c\gamma}{\omega_c + Q_c\gamma}} \quad Q_c = -\frac{2\omega_c L}{c\left(1 - R_1 R_2\right)}$$

ω_0 is the central frequency of our lasing material, γ is its linewidth. ω_c is a nearby cavity mode and Q_c is the cavity quality factor.

The expression for 'mode pulling' allows us to calculate the actual lasing frequency of a laser system based on the parameters of the cavity and those of the lasing material. In the case of the practicum laser, was there mode pulling? Include in your answer a description of mode pulling, where it is likely to be found, and why or why not we should see it in the practicum laser (3pts).